

## Part III

### Chapter 14

#### Series of objects

##### 14.1 Introduction

Objects that engage in indefinite interactions ( introduced in Chapter 3 ) must impose indefinite relations. A system in which an object imposes an indefinite relation may be represented by an indefinite series of objects, where each object imposes a part of the indefinite relation. The series of objects can always be expressed as a series of objects that maintain the same relation which is repeatedly applied. This is called a **recurrence relation**. Recurrence relations also help to provide a realizable form for an indefinite relation.

Indefinite interactions can extend indefinitely in two directions, but they are often assumed to have a starting point, or at least a bound. Sometimes there is a constraint on the indefinite relation and this is done by imposing a bound. This leads to the concept of a **bounded recurrence relation**, which is an infinite series, with one defined boundary. The **bounded sequenced recurrence relations** describe a causal series of objects which has a defined beginning.

If there is a pattern to the relations in a series there is a compact way way of expressing the indefinite relation. The **periodic series** provides a compact form for representing periodic series of relations.

When each of the relations in a series is connected to the rest of the system, they will influence the rest of the system. A **state machine** is a series that can interact with other objects and can be used to describe a sequence of interactions between a series and the rest of the system. A **clock** provides a periodic pattern of interactions to the rest of the system and can thus be seen as a specialization of the state machine.

In some cases, particular kinds of recurrent relations may be replaced by a non-recurrent counterpart. These are called **stable relations**. They can provide a helpful simplification during the analysis of the system.

Two series of objects may interact. Through these interactions, each series will influence the pattern of interactions in the other series. A **synchronized object** is a series that interacts with another series. A

**semaphore** provides a means of synchronizing objects that deal in indefinite alphabets. A **synchronous state machine** provides a state machine that is synchronized to another object.

Editorial: The ideas behind this chapter are important because most digital systems are thought of as systems that engage in a series of interactions. It is for this reason that the current chapter has been included, despite the fact that it is currently being developed. In its current form it should be treated merely as a guide to the potential content. The first priority for revision of Part III is to complete this chapter.

## 14.2 Reference section

The manual pages in this chapter contain the provisional description of the following concepts:

- Recurrence relation
- Bounded recurrence relations
- Bounded sequenced recurrence relation
- Periodic series
- State machine
- Clock
- Stable relation
- Synchronized objects
- Semaphore
- Synchronous state Machine

Editorial: Once these entries have been completed and corrected, such concepts as cardinality preserving and sequence preserving relations will be added.

**NAME**

Recurrence relation

**PURPOSE**

To provide a compact and realizable form for representing indefinite relations.

**SYNOPSIS**

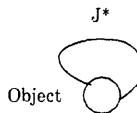
A series of objects that have relations which are repeated periodically can be represented as a single object with a relation that is defined recursively.

Although an indefinite series of objects is unrealizable, it is often convenient to model large series as though they were indefinite. A preliminary analysis can then be simplified.

**CANONICAL FORM**

Figure 14.1 shows an object defined by a recursive relation interacting with an indefinite interaction.

**Figure 14.1 A recurrence relation and an indefinite interaction**



**SPECIFICATION**

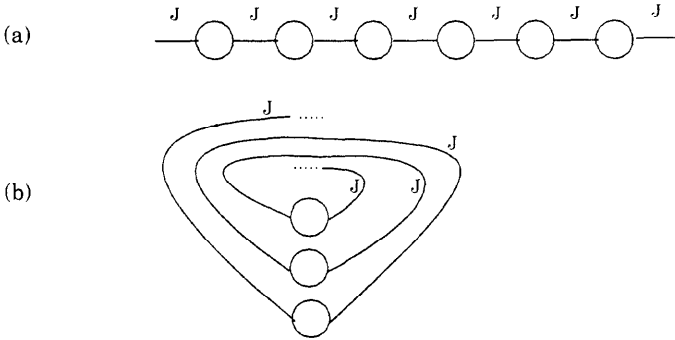
Figure 14.2(a) shows an incomplete diagram of an indefinite series of interacting objects. It is assumed that the alphabet of interaction between the objects is the alphabet J. The relationship maintained by the objects is R.

Figure 14.2(b) shows how the objects can be rearranged so that the connections between them appear in a parallel clusters carrying the same alphabet. The set of parallel connections each carrying a member of the alphabet J can be replaced by a single connection carrying an indefinite combination of the alphabet J

...  $\times J \times J \times J \times J \times J \times J \times J \times J \times \dots$

which can be written as  $J^*$

**Figure 14.2 An incomplete indefinite series (a), and an alternative representation (b).**



The relation maintained by the objects can be written in a similar form. If the relation maintained by each object is  $R$ , then the relation maintained by the set of objects can be written as  $R^*$  and defined recursively as

$$R^* = R \times R^* \cap J \times R \times R^*$$

**SEE ALSO**  
INDEFINITE INTERACTION

**FUTURE DIRECTIONS**

**NAME**

Bounded recurrence relations

**PURPOSE**

To describe a series of objects that begins at a defined boundary.

**SYNOPSIS**

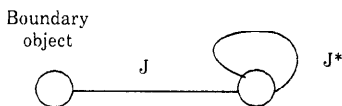
A bounded recurrence relation is a recurrence relation that represents a series of objects that extend indefinitely in one direction only.

Although an indefinite series of objects is unrealizable, it is often a convenient to model large series as though they were indefinite. A preliminary analysis can then be simplified.

**CANONICAL FORM**

Figure 14.3 shows an object that interacts using an indefinite alphabet. An additional object, the boundary object, is included to represent the defined start of the series.

**Figure 14.3 A recurrence relation with a bound**



**SPECIFICATION**

All, but one, of the objects in Figure 14.4(a) impose the same relation,  $R$  say. Without the boundary object the relation maintained by the series of objects can be defined recurrently

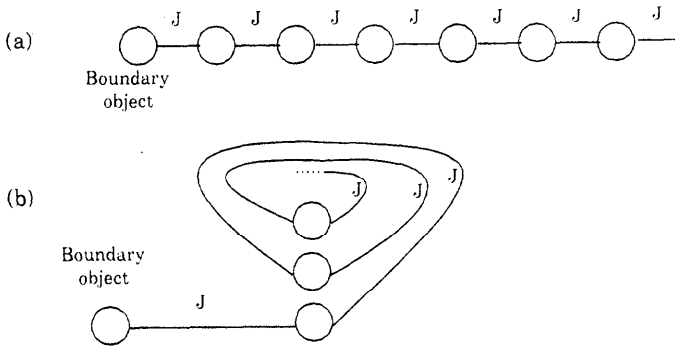
$$R^* = R \times R^* \cap J \times R \times R^*$$

The boundary object places a constraint on this relation, so that if the boundary object imposes the constraint  $J_1$  then the relation imposed by all the objects in Figure 14.4(a) can be written as

$$R \times R^* \cap J_1 \times R \times R^*$$

Editorial: more explanation needed here.

**Figure 14.4 An incomplete and bounded recurrence relation (a),  
and an alternative form (b).**



The constraint imposed by the boundary object *initializes* the recursion.

**REALIZABILITY ISSUES**

**SEE ALSO**  
RECURRENCE RELATION

**FUTURE DIRECTIONS**

**NAME**

Bounded sequenced recurrence relation

**PURPOSE**

To describe a series of objects that begins with a defined beginning.

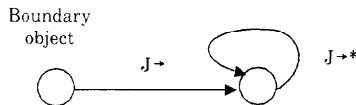
**SYNOPSIS**

A bounded sequenced recurrence relation is a recurrence relation that represents a directed series of objects that extend indefinitely.

**CANONICAL FORM**

Figure 14.5 shows an object that interacts using an indefinite sequenced alphabet. An additional object, the boundary object, is included to represent the defined start of the series.

**Figure 14.5 A recurrence relation with a bound**



**SPECIFICATION**

All, but one, of the objects in Figure 14.6(a) impose the same relation, R say. Without the boundary object the relation maintained by the series of objects can be defined recurrently

$$R \rightarrow^* = R \rightarrow R^* \cap J \rightarrow R \rightarrow R^*$$

The boundary object places a constraint on this relation, so that if the boundary object imposes the constraint  $J_1$  then the relation imposed by all the objects in Figure 14.6(a) can be written as

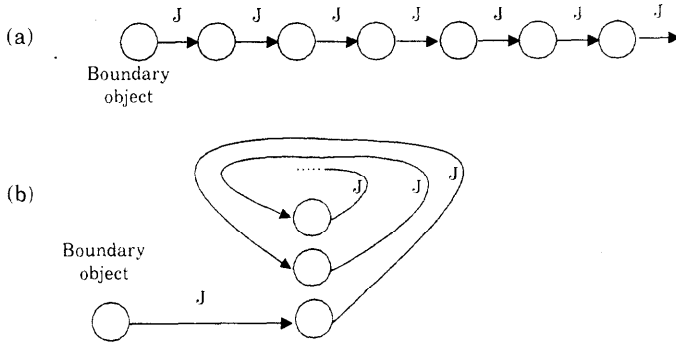
$$R \rightarrow R^* \cap J_1 \rightarrow R \rightarrow R^*$$

Editorial: more explanation needed here.

The constraint imposed by the boundary object *initializes* the recursion.

**REALIZABILITY ISSUES**

Figure 14.6 An incomplete and bounded sequenced recurrence relation (a), and an alternative form (b).



SEE ALSO  
RECURRENCE RELATION

FUTURE DIRECTIONS



**NAME**

Periodic series

**PURPOSE**

To provide a compact form for representing periodic series of relations.

**SYNOPSIS**

A periodic series has a repeating pattern of objects. The repeated pattern need only be described once and an indication given that it is repeated. This can be done using the notation for indefinite alphabets and indefinite relations.

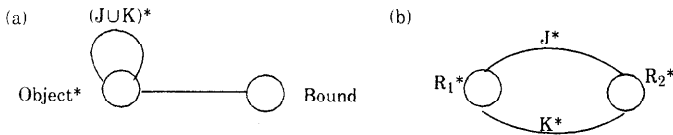
Within a period, provided the alphabets are distinguished, a series can be reduced to a series of objects that all maintain the same relation and interact with the same alphabet. This can result in a useful simplification in the description of the series.

The simplification removes the distinction between the objects in the series. The possible pattern of interactions is maintained but its relative position is undefined. Sometimes the position of a pattern is important so an additional constraint is required to provide the positional information.

**CANONICAL FORM**

Figure 14.7(a) shows an object that provides an indefinite interaction from a unified alphabet. The indefinite interaction that is exhibited on the re-entrant connection is constrained by a bound.

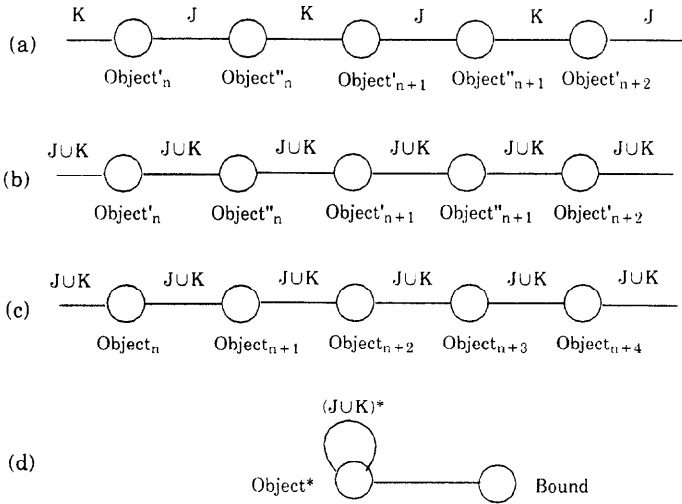
**Figure 14.7 A periodic series with a bound (a), and an alternative canonical form (b)**



**SPECIFICATION**

Figure 14.8(a) shows a periodic series of objects. The interaction alphabets alternate between J and K, and the relations maintained by the objects alternate between R' and R".

**Figure 14.8 A set of diagrams showing the development of the canonical form.**



$$R' \subseteq K \times J$$

and

$$R'' \subseteq J \times K$$

The two alphabets have no symbols in common

$$J \cap K = \{\}$$

Figure 14.8(b) is a diagram in which the alphabets of Figure 14.8 (a) have been replaced by an alphabet JUK. The alphabet allows a symbol for the alphabet J or a symbol from the alphabet K to appear on the connection. A symbol from one of the alphabets would exclude a symbol from the other alphabet.

The relations R' and R'' ensure that symbols from the correct alphabet appear on the corresponding connections in the two diagrams.

In Figure 14.8(c) the distinction between the objects has been removed and the relation maintained is a combination of the two relations R' and R''. The relation, R, maintained by an Object in the figure can be written as

$$R \subseteq (JUK) \times (JUK) = J \times J \cup J \times K \cup K \times J \cup K \times K$$

Because adjacent connections cannot carry the same alphabet the terms taken from the alphabet  $J \times J$  or  $K \times K$  are not allowed and the relation is limited by

$$R \subseteq ((JUK) \times (JUK)) \cap (J \times J \cup K \times K) = J \times K \cup K \times J$$

and is given by

$$R = R' \cup R'' \subseteq K \times J \cup J \times K$$

Although the pattern of interactions in Figure 14.8(c) will be defined in the same way as the pattern in Figure 14.8(a) and 14.8(b) its absolute position in the series is now ambiguous. An additional corresponding constraint, or bound, is required to fix the absolute position of the pattern. This must be provided through one or more interfaces to the series.

The multiplicity of connections carrying the alphabet JUK can be replaced by a single connection carrying the alphabet (JUK)\* and the periodic pattern of objects can be redrawn, as shown in Figure 14.8(d) as a single object.

JUK

has been replaced by a single connection with an indefinite alphabet (JUK)\*

and the series of objects has been replaced by a single object, Object\*. The relation maintained by Object\* is based on the relation (R'UR'')\*

but also links in the bound that is essential to fix the absolute position of the interaction pattern.

## REALIZABILITY ISSUES

Series  
of  
Objects

*ANSA Reference Manual*

PERIODIC  
SERIES

**SEE ALSO**  
INDEFINITE INTERACTION

**FUTURE DIRECTIONS**

**NAME**

State machine

**PURPOSE**

To describe a sequence of interactions.

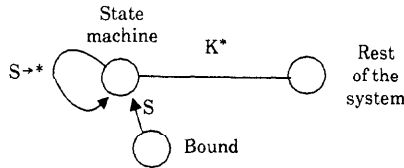
**SYNOPSIS**

A state machine can be represented by a bounded series of objects that interact with the rest of the system. Each of the objects corresponds to a state.

**CANONICAL FORM**

The state machine shown in Figure 14.9 interacts with the rest of the system with an indefinite interaction indicated by the label  $K^*$ .

**Figure 14.9 A state machine interacts with the rest of the system**



**SPECIFICATION**

Assume a bounded sequence of objects. The connections may be labelled with the interactions that are taking place, rather than with the alphabets. The interactions may then form periodic patterns.

If all the objects in the bounded sequence are a part of the state machine and all the objects, except the boundary object maintain one of the relations from a set of relations

**R**

and there is a set of alphabets

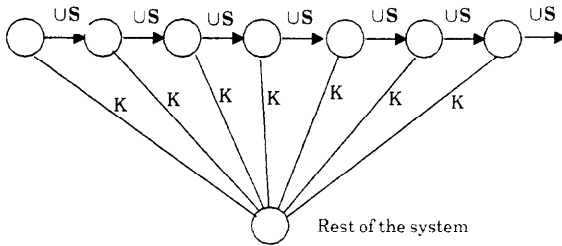
**S**

that correspond to the possible sets of symbols on the interconnections between the objects then the succession of connections can be replaced by connections that carry the alphabet

formed from the union of alphabets included in  $S$ , which is written as  $\cup S$ , as shown in Figure 14.10. Similarly the objects in the series can be given the same relation which is formed from the union of the relations in the set  $R$ , which can be written as  $\cup R$

Each relation is also modified to allow for a connection from the series to the rest of the system.

**Figure 14.10** The state machine can be represented by a sequence of identical objects connected to the rest of the system.



The actual set of symbols that is forced on a connection in the sequence by prior interactions in the series and by the current interaction with the rest of the system is called a representation of the state.

## REALIZABILITY ISSUES

### SEE ALSO

BOUNDED SEQUENCED RECURRENCE RELATION

### FUTURE DIRECTIONS

The specification will be expanded.

**NAME**

Clock

**PURPOSE**

To provide a periodic pattern of interactions

**SYNOPSIS**

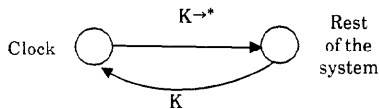
A clock can be represented by a state machine that displays a periodic sequence of states and which interacts with the rest of the system. The constraints imposed on the rest of the system by the sequence of objects in the clock are repeated periodically.

A clock only allows a sequence of a particular period. The position of this pattern in the sequence depends on the bound that is imposed on the clock.

**CANONICAL FORM**

The clock shown in figure 14.11 interacts with the rest of the system with an indefinite sequenced interaction indicated by the label  $K \rightarrow *$ . The clock can be initialized by imposing a constraint on the directed connection labelled  $K$ .

**Figure 14.11 A clock interacts with the rest of the system with a sequenced interaction and can be initialized.**



**SPECIFICATION**

**REALIZABILITY ISSUES**

**SEE ALSO**

STATE MACHINE

**FUTURE DIRECTIONS**

Provide specification.

**NAME**

Stable relation

**PURPOSE**

To provide a compact form for representing a stable series of relations.

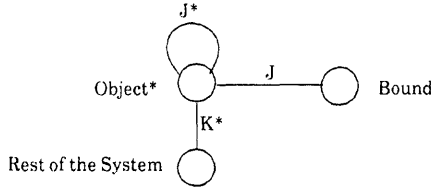
**SYNOPSIS**

A recurrence relation that is built from a reflexive relation may be replaced by its non-recurrent part.

**CANONICAL FORM**

Figure 14.12 shows an object that provides an indefinite interaction. The indefinite interaction that is exhibited on the re-entrant connection is constrained by a bound.

**Figure 14.12 A stable relation that is bounded and connected to the rest of the system**



**SPECIFICATION**

Editorial: The nature of the relation R must be expressed more clearly.

The object labelled Object\* maintains a bounded recurrent relation over members of the alphabets J and K. This may be illustrated as in Figure 14.13, where Object\* has been shown as an incomplete series of objects.

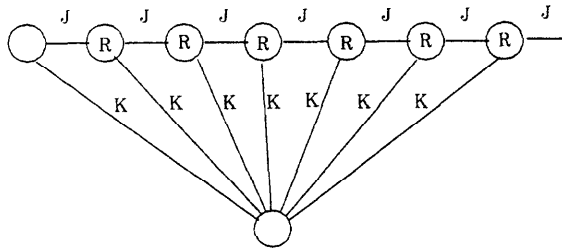
If the relation R is reflexive on J, then

$$R \supseteq K \times I_{jj}$$

and the interactions on the connections labelled J must therefore all be the same. The interactions on the connections labelled K are thus dependent only on the bound and the relation between J and K.

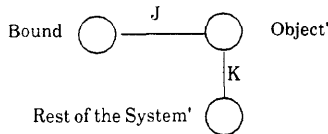


**Figure 14.13 An incomplete series to replace the bounded recurrence relation**



The canonical form can be reduced to a non-recurrent form, as illustrated in Figure 14.14, where the multiplicity of connections between the series and the rest of the system can be replaced by a single connection to a modified system. This is possible because the constraints on each of the connections in the original diagram were identical.

**Figure 14.14 Reduced canonical form for reflexive R**



**REALIZABILITY ISSUES**

**SEE ALSO**

INDEFINITE INTERACTION

**FUTURE DIRECTIONS**

Improve the specification.

**NAME**

Synchronized objects

**PURPOSE**

To ensure that interactions between objects in a series are influenced by the pattern of interactions between objects in another series.

**SYNOPSIS**

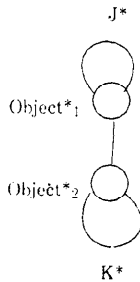
Synchronized objects are objects that represent a series and which are connected to other objects that represent a series by a connection that carries an indefinite alphabet. One series can thereby influence the pattern of interactions of other series.

Certain specializations are possible where the interfaces of the connections are directed.

**CANONICAL FORM**

The canonical form, as illustrated in Figure 14.15, consists of two recurrence relations connected so that they may influence one another and thus be seen as two synchronized series of objects.

**Figure 14.15 The canonical form.**



**SPECIFICATION**

Series  
of  
Objects

*ANSA Reference Manual*

**SYNCHRONIZED  
OBJECTS**

**REALIZABILITY ISSUES**

**SEE ALSO**

**FUTURE DIRECTIONS**

Provision of the specification and examples for the end of the chapter.

**NAME**

Semaphore

**PURPOSE**

A semaphore provides a means of synchronizing objects that deal in indefinite alphabets.

**SYNOPSIS**

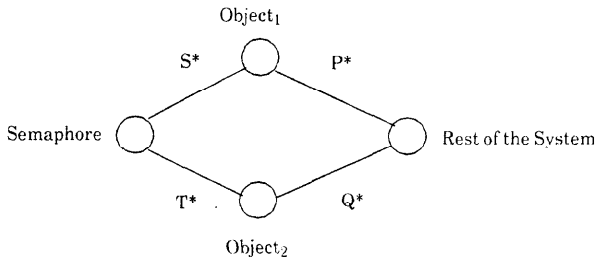
The semaphore provides an indefinite series can be used to synchronize other objects. The synchronized objects may deal in indefinite alphabets but may not be simply described by a single series, or state machine.

The synchronization is frequently aimed at avoiding conflict when interacting with the rest of the system.

**CANONICAL FORM**

The semaphore is represented by an object in object diagrams. The semaphore is connected to several (at least two) objects. Those objects are synchronized by virtue of their connection with the semaphore. The objects are also connected to the rest of the system. From the point of view of the rest of the system the semaphore is invisible, yet the objects are synchronized. The canonical form is shown in Figure 14.16.

**Figure 14.16 A Semaphore**



**SPECIFICATION**

**REALIZABILITY ISSUES**

**SEE ALSO**

**FUTURE DIRECTIONS**

Provision of specification.

**NAME**

Synchronous state Machine

**PURPOSE**

To provide a state machine that is synchronized to another object.

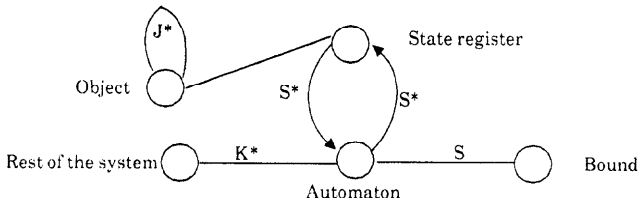
**SYNOPSIS**

Editorial: The current entry is a place holder only

**CANONICAL FORM**

A synchronous state machine consists of two parts. An automaton is an object that interacts with the rest of the system using a composite indefinite alphabet. The other part of the state machine is the state register. The state register is synchronized with another object, which may be part of the rest of the system.

Figure 14.18 A state machine synchronized to another object.



**SPECIFICATION**

**REALIZABILITY ISSUES**

**SEE ALSO**

**FUTURE DIRECTIONS**

Provide full entry.

### **14.3 Examples**

To be provided.